A Life in Operational Research

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Introduction

It has been suggested that as one of the ‘second generation’ of people in OR I write up some of my experiences. The ‘first generation’ were those who took part in wartime OR and the postwar practice of OR, particularly the large nationalised industries. These experiences have been well documented in volumes such as [23], [15] and [10]. I also mention some of this in [27]. Clearly my generation were too young to have participated in this having been born in the war and brought up in the postwar austerity of the 1950s, although one did not recognise this at the time. This was a distinctive phase in the UKs postwar history. The development of postwar OR up to the present has, in many ways, mirrored the political changes from the large nationalised corporations with their impressive OR groups (The National Coal Board, British Steel, British Rail, British Airways etc) and British Petroleum, Shell, Unilever etc. to privatisation, smaller groups and outsourcing of OR to management consultancies (often using other names than ‘OR’). Combined with this has been the vast expansion in Higher Education with OR becoming an established academic subject.

This is a very personal account of my introduction to OR, participation in it and reflections on its successes and failures. Hence it is written in the first person.

Early Experiences

Having been born and brought up in the Camborne-Redruth mining region of Cornwall I realise that a lot of my experiences have been the result of this and, in particular, the Red River that flows down through the centre of the region, for centuries discharging mine waste, in a red plume, into the sea off the north Cornish coast at Godrevy. My great grandmother was a ‘tin streamer’ in the river works there as a girl. It was bright red because of the iron ore impurity which was separated from the tin and copper mined.

Besides the mines (now all closed) the local industries (very close to the Red River) arose to service the mines. In particular Holman Brothers (where my grandfather worked), the main Camborne employer with branches worldwide which manufactured compressed air equipment to power rock drills, which they also made. Bickford Smiths (where my grandmother worked) manufactured explosives for blasting out rock in the mines. Most of my relations and ancestors, at one time, were miners, balmaidens (breaking up the ore), tin streamers (recovering tin from the waste in the Red River) or worked for one of these companies.

Educated at the local boy’s grammar school at Redruth I became obsessive about Mathematics (and Hurdling). Studying Mathematics at Cambridge my two major objectives where (a) seeking ultimate truth though mathematical results and (b) applying mathematics to making British industry more efficient. The first of these led me further to study Mathematical Logic and the second to pursue OR. Unfortunately neither of the objectives has been, more than partially, realised.

At Cambridge in the early 1960s I was very interested in Politics and believed in Harold Wilson’s ‘White Hot Revolution’. I remember meeting and reading the book ‘Decision in Government’ by Jeremy Bray, a mathematician and minister in Wilson’s government who said, among other things that he thought the boardrooms of the future would be filled my members of the Royal Statistical Society and the Operational Research Society and in his book [5] that one of the places to look for the shape of future management methods was the OR Society . My belief in this was somewhat naïve and it never came about. Some years later another politician (one of the ‘Gang of Four’) told me that ‘mathematicians don’t make good politicians’, citing his ‘friend’ Jeremy Bray (but probably also giving me some ‘advice’).

Leaving Cambridge I was fortunate to study for a PhD in Mathematical Logic at Leicester University under the late Professor R.L.Goodstein (now famous for the ‘Goodstein Numbers’, a concrete illustration of Godel’s celebrated incompleteness result). Although a very pure mathematician Goodstein’s ‘Constructive’ approach to mathematics led naturally into Computing.

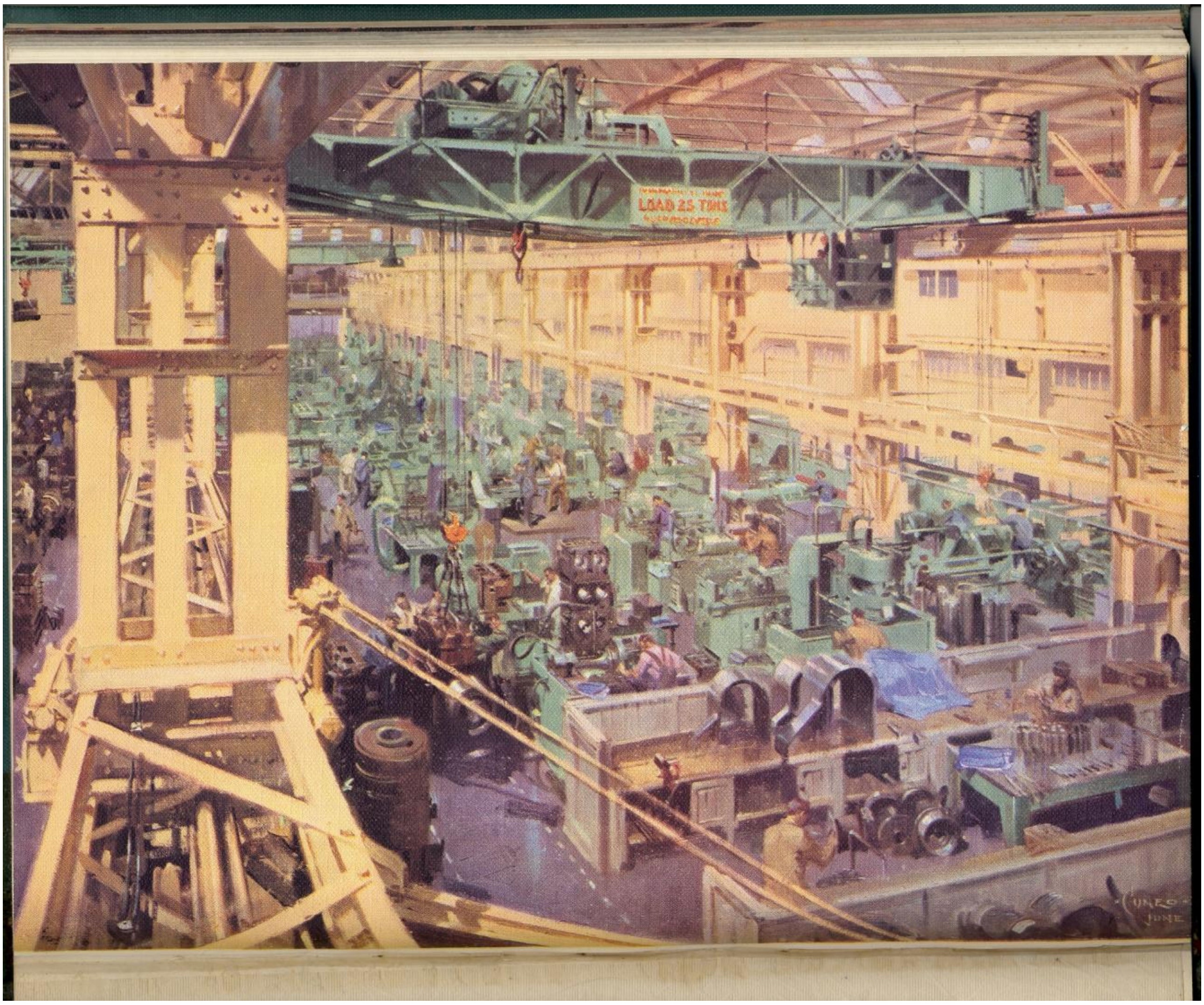
During this time I had a series of vacation jobs back at my home which reinforced my belief in OR. The first was as a labourer on a tin dredger off the coast near Godrevy. Over many centuries the Red River waste had been deposited on the sea bed and it was realised that the sand would contain a lot of valuable tin. A South African mining company hired an old fishing boat (called the ‘Shamrock’) and a series of pipes were lowered over the side to suck up tin from the sea bed. This was then refined in a centrifuge to discover what proportion of the sand would yield tin. The operation went on for a year, sometimes in very rough seas with the pipes bending and breaking and it being difficult to work out one’s exact position. Simple calculations revealed to me that commercial extraction could not possibly be economic but I was told that ‘there is no room for intellectual content in this job’. After a year the enterprise packed up. There would have been great scope for some elementary OR which would have saved a lot of money. But it provided me with employment. I have watched the Shamrock rotting in Hayle Estuary over forty years. The keel is all that now remains.

The Red River at Godrevy about 1970

The Shamrock in Hayle Estuary in the 1960s

A more productive vacation job was at Holman Brothers in Camborne. A far-sighted ‘Forward Planning’ senior manager had suggested I build a Linear Programming ‘Product Mix’ model of their manufacturing operations to try to connect their production and marketing activities. This had been motivated by their aquiring an IBM360 computer which was being largely used for payroll purposes. We built the model with some success and it revealed where productive capacity was most tight and, to the directors’ ‘astonishment’, that some of the production was going on off-site through limited capacity. Looking back on this model I now realise that the use of a computer, (we used an LP package called ‘CAPLINE’) was quite unnecessary. A simple analysis would have shown as much. But the abstraction involved in looking at the total organisation was a highly valuable exercise. It would not have been done if they had not purchased a computer in the first place.

During this job I was also encouraged to look at the Job Shop in ‘No. 1 works’.
This was where my grandfather had worked as a Fitter and Turner in the past. It is illustrated below.



No 1 Works at Holman Brothers

The works was almost a Flow Shop and I remember devising some simple rules for scheduling the jobs which I later realised were ‘Johnson’s Rules’ and had been discovered before. The machines , the Grinders, Twin Head Borers, Capstans, Duplex Mills, Drills etc still stick in my mind. The scheduling aspect was a subproblem of the Product Mix problem for which we had built an LP model. However I soon realised I was only looking at one aspect of the problem and ignoring other aspects, such as its stochastic and ‘online’ nature (jobs were often ‘scheduled’ by ‘progress chasers’ looking after a particular order), and its relations to the wider problems. This has coloured my attitude to Job Shop Scheduling, with which I remained fascinated for some time. Since then a whole ‘industry’ has grown up among academics looking at more and more ‘pure’ variants of job shop scheduling and ignoring the real problems. One suspects that most of them have never been on a factory floor such as that illustrated. As a former colleague (a Pure Mathematician) once said to me ‘this is not OR, it is *boring* Pure Mathematics’.

Another problem which I looked at concerned ‘optimising’ the design of ‘logic circuits’ which a subsidiary of the firm (Maxam Power) produced. They manufactured ‘NOR gates’ which were devices depending on the flow of air (instead of electricity). These devices had two inputs of air flow and only produced an output if neither of the inputs were on (ie the ‘NOR’ function). Since ‘NOR’ is a Complete Connective of Boolean Algebra these devices could be connected together to perform any logical function. The aim was to minimise the number of gates needed to produce devices which could be used in conditions of high temperatures where electrical systems would fail. Although I did not manage to achieve the optimum I produced some economical solutions using the method of Quine [21]. All this connected with my PhD in Mathematical Logic. I later wrote it up in [24] and much later formulated the problem as an Integer Programmme in [32].

The Forward Planning Manager who had hired me spent a lot of his time trying to get the company to create new markets in Europe. The current markets were historically based on metal mining regions around the world to which Cornish miners had emigrated eg South Africa, Rhodesia, Australia etc. But Cornwall was not a good place to do heavy manufacturing given its (then) poor transport links. The company soon experienced a severe downturn in its orders. The Forward Planning Manager left and no more OR took place. The factory and job shop have long been demolished and are now the site of a Tesco supermarket. Holman Brothers was Camborne’s main employer with thousands working there.

Early Academic Research

Although classified as ‘Pure Mathematics’ the research I did for my PhD in Mathematical Logic was applicable to problems of both continuous and discrete optimisation. I looked at Constructive methods of solving parts of mathematics. Given that the full system of Arithmetic is undecidable (there can be no universal algorithm for deciding the truth or falsity of any statement) one focuses attention on ‘fragments’. One such was the ‘Theory of Dense Linear Order’ (Real numbers with the ‘<’ and ‘=’ relations and the Predicate Calculus). I realised that Linear Programmes could be formulated within this structure and solved as such by a procedure due to Langford [16]. I programmed this in FORTRAN with little success. It was some time later, when working for IBM, that on a visit to the USA Phil Wolfe thought he had seen a similar method before and found it in George Dantzig’s book [11]. It had been proposed by Fourier in 1836 [14] as a method of solving inequalities. I have spent much time subsequently refining it and applying it, with some success, to special structures and eventually wrote it up in [30]. At this time I also found another Decision Procedure due to Presburger [20] for Arithmetic without multiplication and realised this could be applied to solving Integer Programmes. I wrote it up later in [26] and produced a dual version [29].

Industrial Work

My first full-time employment was with IBM(UK) in London. There I joined a small team (known as 'Applications Development') working on the development of Operational Research software. Firstly I worked on a system (PMS) for Resource Allocation on a PERT Network using IBM360 computers. This used heuristics to obtain suboptimal solutions to this combinatorially difficult problem. Then I worked on developing the MPSX Mathematical Programming System for the IBM370. In particular I worked on the REDUCE system which enabled models to be reduced, often dramatically, in size by computationally quick and simple operations. This was prompted by observing that many (most?) commercial models contained large redundances and/or were infeasible or unbounded. All this could be sorted out at little computational cost, enabling the models to be made much smaller, more transparent and correct. I also realised that similar procedures could be applied to the Dual model and the effect of reducing the Primal and the Dual in combination with each other could be dramatic and tell one a lot about the situation being modelling (see [6]). Unfortunately the Dual addition was never implemented. Later I implemented a better system for the ICL2900 system. This work was prompted, in part, by the fact that British Petroleum had a notoriously difficult 822 Constraint model linking together 6 oil refineries in Europe. All the computer companies had difficulty solving it in a reasonable period of time (today it would be trivial to solve). REDUCE had little effect on it. The truth finally emerged that they had reduced it already. Therefore it consisted of the 'difficult' part of a much larger model. Today reduction methods (usually known as PRESOLVE) are routinely incorporated into most commercial systems.

The major contribution I made at IBM was in the development of Integer Programming (IP) with particular reference to modelling. Among the projects I participated in was a worldwide survey of the use of LP and IP among IBM customers. This persuaded me that Modelling was a much neglected aspect of the subject. It was possible to model IPs in different ways, some of which were much easier to solve than others. Much of this was not understood. Two early papers were [25] and [28]. In addition little attention was paid to 'user friendly' modelling systems. IBMs input format (known as MPS Format) was the industry standard and had been created by mathematicians thinking in terms of matrices, not with the user in mind. Since then the development and use of modelling languages has become the norm. In those days 'models' took the form of large boxes of punched cards which were carted around to run on large mainframe computers. It was before the days of interactive computing, emails and the internet. In retrospect the computational problems of solving LPs and IPs assumed too much relative importance compared with the problems of creating, building and interfacing. There was much competition among computer companies and consultancies such as IBM, ICL, CDC and SCICON with very substantial packages such as MPS(X), LP2900, SCICONIC/ UMPIRE, OPHELIE, and APEX to solve larger and larger models more quickly than each other. Also while at IBM I realised that a major defect of the MPS system was its INVERT procedure which did not use L\U Decomposition, but I did not persuade them to change it. The packages represented huge capital investments and had to be run on large mainframe computers. The dramatic advance in the ability to solve large models (and therefore the decline in the research effort into computational methods) is one of the big successes of OR. Although IP and Stochastic Programming still remain a computational challenge. An LP with a billion variables and a million constraints was recently solved at Edinburgh University. Now the most widely used packages (such as CPLEX and Xpress) run on laptops and have interfaces to Spreadsheets and Databases). What is now needed is the ability to build, understand maintain and incorporate models into large Decision Support Systems. A lot of my career, then and subsequently, has been spent building commercial models for applications such as food blending, distribution, manpower allocation and production.

Academic Research

While it is often difficult to separate commercial from academic research in OR much of my work from the 1970s onwards has been in academia.

At Edinburgh University much time was spent designing and implementing modelling languages with Robin Day and Ken McKinnon ([12] and [18]) using algebraic notation and the Predicate Calculus. Although all commercial systems now use such languages I feel there is more to be done here particularly regarding the second of the above papers.

On a theoretical levelI I have refined and enhanced both the Primal and Dual forms of the Fourier-Motzkin elimination method to give ,what I believe are, clearer ways of understanding fundamental results in LP (eg the Duality theorem and the result that if there are optimal solutions there is always a basic one). Also I have had some success in specialising the methods to particular applications (eg DEA (with Gautam Appa [3]), Complementarity and Game Theory) to produce, alternative models, computational improvements and more information than that produced by traditional iterative methods.

I am still searching for ‘the Dual of an Integer Programme’ which some might believe does not exist. But I believe its absence represents a gap in our understanding of Discrete Mathematics. A number of partial solutions exist but lead to computational and representational problems. More importantly they lack ‘transparency’ which makes them difficult to explain to the non-specialist. Such a requirement is vital to the successful implementation of models and methods in OR. A particular application would be to allocating fixed costs in a manner which leads to an optimal allocation of resources. This problem is recognised by Accountants and Economists as ‘unsolved’. I have to admit to a failure, so far, to bridge the gap between the Accounting and OR ways of tackling the problem. I discuss the problem in [8] and [31].

This is connected to the related problem of allocating resources or (fixed) costs in a ‘fair’ manner (see [7]). Such problems can, of course, be made optimisation problems if one’s objective is made a Maximin one (assuming one accepts the Rawlsean notion of fairness) (see eg Rawls[22] and Binmore[4]). The problem of reconciling economic efficiency with a fair allocation of resources is a central one in any democratic society. The first is tackled by Economists and the second by regulatory Accountants. But they tend not to use concepts from Mathematical Programming. I do feel we need to speak the same language and build common concepts as was done by the pioneering mathematicians and economists who developed Linear programming and Game Theory in the 1940s and 1950s (eg Dorfman, Samelson and Solov [13]). Such links have gone. A particularly important relevant area is to resource allocation in the Health Service. QALYs (Quality Adjusted Life Years) can be used by Mathematical Programming to maximise utility or equity (by a Maximin objective). Connected to this is the use of Game Theory (which like QUALYs arguably emerged from OR but are not now usually recognised as such). The Nucleolus of a Cooperative Game uses a Maximin criterion and has been applied to allocating common costs in a 'fair' manner (see eg Peyton Young [34]).

A 'classic' problem of Integer Programming (IP) is the Travelling Salesman Problem (TSP). Although dramatic advances have been made by the combined use of more powerful computers and Polyhedral Combinatorics it remains a stubborn problem (see [17] and [1]). I have done work on alternative formulations as an IP but not produced a 'better' formulation (Orman and Williams [19]). Like IP in general I feel a major conceptual breakthrough in Discrete mathematics is needed. All our methods, to date, rely on the sophisticated application of simple ideas (relaxations, cuts and partial enumeration). For many applications (eg Job Shop Scheduling) a Computational Complexity classification has moved from being a secondary objective of academics to being the primary one. While bearing some relation to computational difficulty it is only one measure and has, in my opinion, diverted attention from the real problems. The use of Constraint Logic Programming (CLP) has been very successful with some IP models (but not the TSP) but relies on clever computer implementations (and the fact that computers are dramatically faster) and data structures rather than much mathematics (and little Logic) (see Wilson and Williams [33]), although it can be seen as akin to a PRESOLVE procedure. This is not, of course, to be decried. OR is about solving problems. Unfortunately the hope of fully integrating CLP with IP has not been fully realised for general computer systems, only for specific applications (eg Bandwidth selection).

There are important applications of extensions of the TSP (eg the 2-Period TSP on which I worked with my research student , Martin Butler, with some success) [9]). A challenging, more recent one, is to Genome sequencing (see in eg [2])

The Organisation of OR and Reflections on its State

OR has not lived up to the expectations generated in the 40s, 50s and 60s. Much of the theoretical work has migrated into other 'disciplines' such as Economics and Computer Science. While this may not matter very much it has left the subject exposed. The membership of the UK OR Society has remained at about 3000, or below, for all my working life. This is incredible given its economic and social importance. It is now fairly rare for OR academics to have Industrial experience. Therefore we have lost a dimension of reality. The ratio of practitioners to academics at the OR Conference has steadily declined. This is reflected by some of the papers appearing in journals. I remember being present, with about 6 others, when we started COPIOR (Committee Of Professors in OR) at a meeting in Aston in about 1977. We had great aspirations in the early years trying to get ourselves on Royal Commissions. Although there was some success the overall result was not as effective as we would have hoped. Now COPIOR has expanded to nearly 100 and become inward looking, confining its attentions to fighting its corner in national university funding. Nevertheless OR survives which is some achievement but we remain ‘fringe players’.

It seems to me that OR fails to make much impact at the policy level in areas such as Health, Crime and Defence, although it is used at lower levels. Health economists are much more significant. The concept of a ‘Shadow Price’ for ‘sudden’ resource allocation decisions (driven by political expediency) in these areas seems unknown. Performance Targets are set without any prior research into unintended consequencies. OR has the expertise to think more deeply about these matters but does not seem to be called upon (although The Royal Statistical Society has looked at Performance Targets). Accountants (probably because of their statutary role) play a much bigger part in making capital investment decisions than do OR professionals by means of models. When models are used they are usually referred to as ‘Computer Models’ irrespective of their technical nature. The computer is the important component not the nature, accuracy and relevance of the model.

Great misconceptions still exist about OR. I remember one Vice Chancellor telling me that ‘Decision Making under Uncertainty’ was impossible when I was trying to introduce a course of that name! When introduced to a Director of a major corporation, while at Edinburgh University, as a Professor of Management Science I was told by him that ‘Management’ was an ‘Art’ not a Science. Like many others I have failed to get OR accepted where it most matters. And the Red River is no longer red as mining has stopped in Cornwall.

Postscript 2012: The Red River is again red as South Crofty mine has been reopened to extract the rare earth element Indium. The workings are being drained into the Red River.

References

1.Applegate DL, Bixby RE, Chvatal V and Cook WJ (2006). *The Travelling Salesman Problem: A Computational Study,* Princeton University Press

2.Appa G, Pitsoulis L and Williams HP (Eds) (2006) *Handbook of Modelling forDiscrete Optimisation,* Springer

3.Appa G and Williams HP (2006). A New Framework for the Solution of DEA Models, *European Journal of Operational Research, 604-615*

4.Binmore K (2005). *Natural Justice,* Oxford University Press

5.Bray J (1970). *Decision in Government* , Gollancz, London

6.Brearley AL, Mitra G and Williams HP (1975). Analysis of Linear Programming Problems Prior to Applying the Simplex Algorithm, *Mathematical Programming* 8 54-83

7.Butler M and Williams HP (2002), Fairness Versus Efficiency in Charging for the Use of Common Facilities, *Journal of the Operational Research Society,* 53 1324-1329

8.Butler M and Williams HP (2006). The Allocation of Shared Fixed Costs, *European Journal of Operational Research* 170 391-397

9.Butler M, Williams HP and Yarrow L-A (1997). The 2-Period Travelling Salesman Problem Applied to Milk Collection in Ireland, *Computational Optimization and Applications* 7 291-306

10.Coales JF and Tomlinson RC (Eds) (1977). Operational Research and Systems Analysis in Decision Making, *Philosophical Transactions of the Royal Society of London* 287 352-544

11.Dantzig GB (1963). *Linear Programming and Extensions,* Princeton University Press,

12.Day R and Williams HP (1986). MAGIC: The Design and Use of an Interactive Modelling Language for Mathematical Programming, *IMA Journal of Mathematics in Management,* 1 53-65

13.Dorfman R, Samuelson PA and Solov RM (1958). *Linear Programming and Economic Analysis,* McGraw Hill, New York

14.Fourier JBJ (1836). Solution d'une question particuliere du calcul des inegalites, *Oevres II, Paris*

15.Kirby MW (2003). *Operational Research in War and Peace*, Imperial College

Press, London

16.Langford CH (1927). Some Theorems on Deducibility, *Ann. Of Math. I,28 16-40*

17.Lawler EL, Lenstra JK, Rinnooy Kan AHR and Schmoys DB (Eds) (1985). *The Travelling Salesman Problem,* Wiley, Chichester

18.McKinnon KIM and Williams HP (1989). Constructing Integer Programming Models by the Predicate calculus, *Annals of OR: Special Edition on Linkages with Artificial Intelligence, 21 227-246*

19.Orman A and Williams HP (2006). A Survey of Different Integer Programming Formulations of the Travelling Salesman Problem, *Advances in Computational Management Science 9, Optimisation, Econometric and Financial Analysis,* edited by C.Gatu and E Kontoghiorghes

20.Presburger M (1930). Uber die Vollstandigkeit eines gewissen Systems der Arithmetik ganzer Zahlen in welchem die Addition als einzige Operation hervortritt, *C.-R. I Congres des Math. Pays Slaves, Warsaw* 92- 101

21.Quine WV (1952). The Problem of Simplifying Truth Functions, *American Mathematical Monthly,* 59 521-531

22.Rawls J (1972). *A Theory of Justice,* Oxford University Press

23.Waddington CH (1973).*OR in World War II*, Elek Science, London

24.Williams HP (1968). The Synthesis of Logical Nets Consisting of NOR units, *The Computer Journal,* 11 173-176

25.Williams HP (1974). Experiments in the Formulation of Integer Programming Problems, *Mathematical Programming Studies,* 2 180-197

26.Williams HP (1976). Fourier-Motzkin Elimination Extended to Integer Programming
Problems, *Journal of Combinatorial Theory,* 21 118-123

27.Williams HP (1978). *Modelling in Management Science: Is the Model More Important than the Mathematics*, Inaugural Lecture as Professor of Management Science, Edinburgh University

28.Williams HP (1978). The Reformulation of Two Mixed integer Programming

Problems, *Mathematical Programming,* 14 325-331

29.Williams HP (1983). A Characterisation of All Feasible Solutions to an Integer Programme, *Discrete Applied Mathematics 5 147-155*

30.Williams HP (1986). Fourier's Method of Linear Programming and its Dual, *The American Mathematical Monthly* 93 681-695

31.Williams HP (1997). Integer Programming and Pricing Revisited, *IMA Journal of Mathematics Applied in Business and Industry,* 8 203-214

32.Williams HP (1999). *Model Building in Mathematical Programming,* Wiley, Chichester,UK

33.Williams HP and Wilson JM (1998) Connections between Integer Programming and Constraint Logic Programming, *INFORMS Journal on Computing,* 10 261-264

34.Young HP (Ed) (1985). *Methods and Principles of Cost Allocation,* Elsevier

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